Review: Simplifying Fractions

Recall your laws of fractions:

To add or subtract fractions, we require a common denominator then add the numerators keeping the denominator the same:

$$\frac{a}{b}+\frac{c}{d}=\frac{ad}{bd}+\frac{bc}{bd}=\frac{ad+bc}{bd}$$

To multiply fractions, we simply multiply numerators with numerators, and denominators with denominators: $\left(\frac{a}{b}\right)\left(\frac{c}{d}\right)=\frac{ac}{bd}$

To divide fractions, we take the reciprocal of the second fraction, then multiply: $\left(\frac{a}{b}\right)\div\left(\frac{c}{d}\right)=\left(\frac{a}{b}\right)\times\left(\frac{d}{c}\right)=\frac{ad}{bc}$

Example 1:

Simplify the following expression:

$$\frac{\frac{a}{3b}-\frac{b}{a}}{b}+\frac{1}{a}$$

$$=\frac{\frac{a^2}{3ab}-\frac{3b^2}{3ab}}{b}+\frac{1}{a}$$

$$=\frac{\frac{a^2-3b^2}{3ab}}{b}+\frac{1}{a}$$

$$=\frac{a^2-3b^2}{3ab}\times\frac{1}{b}+\frac{1}{a}$$

$$=\frac{a^2-3b^2}{3ab^2}+\frac{1}{a}$$

$$=\frac{a^2-3b^2}{3ab^2}+\frac{3b^2}{3ab^2}$$

$$=\frac{a^2}{3ab^2}$$

$$=\frac{a}{3b^2}$$

Review: Factoring

<u>Recall following equations:</u>

- Difference of two squares $x^2 - y^2 = (x - y)(x + y)$

- Sum of two cubes $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

- Difference of two cubes $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

- Quadratic Formula: Solutions to $ax^2 + bx + c = 0 (a \neq 0)$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Review: Exponent Laws

Recall your laws of exponents:

$$\left(a^b\right)(a^c) = a^{b+c}$$

$$\left(a^b\right) \div (a^c) = a^{b-c}$$ (Provided $a \neq 0$)

$$\left(a^b\right)^c = a^{bc}$$

$$a^0 = 1$$ (Provided $a \neq 0$)

$$a^{-b} = \frac{1}{a^b}$$ (Provided $a \neq 0$)

$$\frac{1}{a^{-b}} = a^b$$ (Provided $a \neq 0$)

$$\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$$ (Provided $a, b \neq 0$)

$$(ab)^c = a^c b^c$$

$$\left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$$ (Provided $b \neq 0$)

$$a^{b/c} = \sqrt[c]{a^b}$$

Some more lesser known and obscure exponent laws:

$$\sqrt{x^2} = |x|$$

$$\sqrt{x^2} = x$$ Provided $x \geq 0$

$$\sqrt{x^2} = -x$$ Provided $x < 0$

Examples: Exponent Laws

Example 2:

Simplify the following expression. Be sure your final answer has no negative exponents:

$$\left(\frac{2ab^{-1}}{6a^2b^{-3}}\right)^{-2}$$

Solution:

$$\left(\frac{2ab^{-1}}{6a^2b^{-3}}\right)^{-2}$$

$$=\left(\frac{a^{-1}b^2}{3}\right)^{-2}$$

$$=\frac{a^2b^{-4}}{3^{-2}}$$

$$=\frac{a^2 3^2}{b^4}$$

$$=\frac{9a^2}{b^4}$$

Examples: Exponent Laws

Example 3:

Simplify the following expression where $x < 0$ and $y > 0$. Be sure your final answer has no negative exponents.

$$3\sqrt{3x^4y} - x\sqrt{12x^2y} + x^2\sqrt{75y}$$

Solution:

$3\sqrt{3x^4y} - x\sqrt{12x^2y} + x^2\sqrt{75y}$

$= 3\sqrt{3x^2x^2y} - x\sqrt{3(4)x^2y} + x^2\sqrt{3(25)y}$ (split into things that can be square rooted)

$= 3(-x)(-x)\sqrt{3y} - x(-x)(2)\sqrt{3y} + x^2(5)\sqrt{3y}$ (since $x < 0$ we have $\sqrt{x^2} = -x$)

$= 3x^2\sqrt{3y} + 2x^2\sqrt{3y} + 5x^2\sqrt{3y}$

$= 10x^2\sqrt{3y}$

Review: Equation of a Line

Recall ideas related to the equation of a line $y = mx + b$

$m \rightarrow$ is our slope and is calculated using $m = \frac{y_2 - y_1}{x_2 - x_1}$

$b \rightarrow$ is the y – intercept, but can also be found by subbing in a point into the equation (as long as you know m)

Two lines are parallel when the have equal slope like $y = 5x - 7$ and $y = 5x - 2$

Two lines are perpendicular when they have negative reciprocal slopes like: $y = \frac{2}{3}x + 5$ and $y = -\frac{3}{2}x - 1$

Example 4:

Find the equation of a line that goes through $(4,5)$ and $(-6,10)$

Solution:

We calculate our slope:

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{10-5}{-6-4}$

$m = \frac{5}{-10}$

$m = -\frac{1}{2}$

$\therefore y = -\frac{1}{2}x + b$

Next we solve for b by using one of the points. Sub in $(4,5)$ we get:

$y = -\frac{1}{2}x + b$

$5 = -\frac{1}{2}(4) + b$

$5 = -2 + b$

$7 = b$

$\therefore$ The equation of the line is $y = -\frac{1}{2}x + 7$

Examples: Equation of a Line

Example 5:

Find the equation of a line that is perpendicular to $y = 3x - 1$ and goes through the point $(5,1)$

Solution:

We calculate our slope:

$m_{\perp} = -\frac{1}{3}$

$\therefore y = -\frac{1}{3}x + b$

Next we solve for b by using one of the points:

Sub in $(5,1)$ we get:

$y = -\frac{1}{3}x + b$

$1 = -\frac{1}{3}(5) + b$

$1 = -\frac{5}{3} + b$

$1 + \frac{5}{3} = b$

$\frac{8}{3} = b$

$\therefore$ The equation of the line is $y = -\frac{1}{3}x + \frac{8}{3}$

Review: Sets

A *set* is an *unordered* collection of zero or more objects.

The objects are called the elements of the set.

For sets, we'll use variables *S*, *T*, *U*, …

The notation $a \in S$ means a is an element of S and $a \notin S$ means a is not an element of S .

For sets, we'll use variables *S*, *T*, *U*, …

The notation $a \in S$ means a is an element of S and $a \notin S$ means a is not an element of S .

Two sets are called equal *if and only if* they contain exactly the same elements.

Symbols for some special infinite sets:
N = {0, 1, 2, …} The natural numbers.
Z = {…, -2, -1, 0, 1, 2, …} The integers.

Q= {…, -1,-0.5, 0, 0.5,1, …} The rational numbers
R = The "**r**eal" numbers, such as 1.123456789…

∅ ("null", "the empty set") is the unique set that contains no elements

Operators on sets

- For sets A, B, their *union* $A\cup B$ is the set containing all elements that are either in A, **or** ("$\vee$") in B (or, of course, in both).

  $A\cup B = \{x \mid x\in A \vee x\in B\}$.

  $\{2,3,5\}\cup\{3,5,7\} = \{2,3,5,3,5,7\} = \{2,3,5,7\}$
- For sets A, B, their *intersection* $A\cap B$ is the set containing all elements that are simultaneously in A **and** ("$\wedge$") in B.

  $A\cap B \equiv \{x \mid x\in A \wedge x\in B\}$.

  $\{2,4,6\}\cap\{3,4,5\} = \{4\}$

Review: Interval Notation

Recall Interval Notation:

[	**used to include** the leftmost point of the interval
(	**used to not include** the leftmost point of the interval
]	**used to include** the rightmost point of the interval
)	**used to not include** the rightmost point of the interval
$\cup$	joins two intervals together
$-\infty$	used to indicate we are going as low as we like
∞	used to indicate we are going as high as we like

Note: It is proper form to write the intervals in increasing numerical order.

Example 6

Write the following using interval notation:

a) $2 \leq x < 5$

b) $x < -3$

c) $x \geq 8$

d) $0 \leq x < 2$ or $x \geq 11$

Solution:

a) $[2,5)$

b) $(-\infty, -3)$

c) $[8, \infty)$

d) $[0,2) \cup [11, \infty)$

Review: Solving Inequalities

<u>Recall ideas related to solving inequalities:</u>

If the inequality only has linear terms (ie $3x - 5 > 7 - 2x$)

1) Isolate x to one side and constants to the other side
2) If you ever multiply/divide by a negative number, switch the direction of the inequality.

If the inequality has other higher degree terms (ie $3x^2 - 5x > 7 - 2x^3$)

1) Bring all terms on one side and simplify the expression
2) Fully factor the expression
3) Determine the zeros of the polynomial in the numerator and denominator (if applicable)
4) Create an interval table of the form:

$-\infty$ 1st Zero 2nd Zero … Last Zero ∞

Factors	$\#_1$	$\#_2$	…	$\#_L$
Factor 1	+ or -	+ or -	…	+ or -
Factor 2	+ or -	+ or -	…	+ or -
…	…	…	…	…
Product	+ or -	+ or -	…	+ or -

5) User the "Product" row in correspondence with the " $>$ " or " $<$ " sign to solve the inequality.
6) If the sign says $\geq$ or $\leq$, then allow for the zeros to be included in your answer from the numerator but you must exclude the zeros from the denominator.

Important note: <u>NEVER multiply or divide by a variable when solving an inequality</u>. Since we don't know if the variable is positive or negative, we don't know if we should switch the signs or not, which can often lead to wrong answers!

Examples: Solving Inequalities

Example 7:

Solve the following inequality: $-5x + 7 > x - 8$

Solution:

$-5x + 7 > x - 8$

$-5x - x > -8 - 7$

$-6x > -15$

$\frac{-6x}{-6} < \frac{-15}{-6}$

$x < \frac{5}{2}$

$\therefore\ x \in \left(-\infty, \frac{5}{2}\right)$ (if we wanted to use interval notation)

Examples: Solving Inequalities

Example 7:

Solve the following inequality: $x \geq \frac{9}{x}$

Solution:

$x \geq \frac{9}{x}$

$x - \frac{9}{x} \geq 0$

$\frac{x^2-9}{x} \geq 0$

$\frac{(x-3)(x+3)}{x} \geq 0$

The zeros are: $x = -3, x = 3$ for the numerator and $x = 0$ for the denominator:

$-\infty$ $\quad -3 \quad$ 0 $\quad 3 \quad$ ∞

Factors	-4	-1	1	4
$x-3$	$-$	$-$	$-$	$+$
$x+3$	$-$	$+$	$+$	$+$
x	$-$	$-$	$+$	$+$
Product	$-$	$+$	$-$	$+$

Thus we have our expression ≥ 0 on the interval $[-3,0) \cup [3,\infty)$. We include -3 and 3 as they are zeroes that appear in the numerator and are defined. We exclude 0 as it is a zero that appears in the denominator and is undefined.

Review:Absolute Value

- The absolute value of a number denoted by $|x|$ is defined as
- $|x| = \begin{cases} x, x \geq 0 \\ -x, x < 0 \end{cases}$
- Properties of absolute value:
- 1. $|-a| = |a|$
- 2. $|ab| = |a||b|$
- 3. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$
- 4. $|a+b| \leq |a| + |ab|$
- 5. $|x| = a \Leftrightarrow x = \pm a$
- 6. $|x| < a \Leftrightarrow -a < x < a$
- 7. $|x| > a \Leftrightarrow x > a \; or \; x < -a$
- 8. $|x| \leq a \Leftrightarrow -a \leq x \leq a$
- 9. $|x| \geq a \Leftrightarrow x \geq a \; or \; x \leq -a$

Example: Absolute Value

a) Solve the equation $|2x - 3| = 7$

Solve the following inequalities expressing the solution sets in intervals or union of intervals. Also, show each solution set on the real line.

b) $|5-\frac{2}{x}| < 1$

c) $|1-x| > 1$